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Spectral Width of Reflection from a Cholesteric Liquid Crystal

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Spectral Width of Reflection from a Cholesteric Liquid Crystal.

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The aim of this note is to rectify an error in the expression for the spectral width of reflection $\Delta\lambda$ from a cholesteric liquid crystal derived in a previous paper.[1] The correct derivation presented here leads to a value of $\Delta\lambda$ in agreement with the de Vries theory.[2]

We first write down the formulae for reflection when light is incident normally on the surface of a non-absorbing anisotropic crystal.[3] Let μ_0 be the refractive index of the first medium from which light is incident and μ_1, μ_2 the principal indices of the anisotropic crystal. If the incident light is of unit amplitude and linearly polarized at an angle θ with respect to the principal axis for which the refractive index is μ_1, the reflected light will consist of two vibrations linearly polarized along the two principal axes:

$$\left.\begin{aligned} E_1 &= -\frac{\mu_1 - \mu_0}{\mu_1 + \mu_0}\cos\theta \\ E_2 &= -\frac{\mu_2 - \mu_0}{\mu_2 + \mu_0}\sin\theta \end{aligned}\right\} \tag{1}$$

Now, the cholesteric structure is regarded as a pile of thin birefringent layers, the principal axes of the successive layers turned through a small angle β. Let the principal axes of the first layer be along OX, OY. If the structure is right-handed, i.e., β is positive, it can be shown[4] that right circular light incident normal to the layers is reflected without change of sense of circular polarization when $\lambda_0 = \mu P$, where P is the pitch, μ the refractive index and λ_0 the wavelength in vacuum. To calculate the reflection coefficient at the boundary between the $(\nu+1)$th and $(\nu+2)$th layers, we resolve the incident light vector along the principal axes of the $(\nu+1)$th layer which are inclined at an angle $(\nu+1)\beta$ with respect to OX, OY. The resolved components are[4]

$$\begin{bmatrix}\xi\\ \eta\end{bmatrix}=\begin{bmatrix}1\\ i\end{bmatrix}\exp[i\{(\nu+1)\beta-\phi_{\nu+1}\}],$$

where $\phi_{\nu+1}=2\pi\mu(\nu+1)p/\lambda$, where p is the thickness of each layer. At the boundary, the ξ vibration emerges from a medium of refractive index μ_1 and the η vibration from a medium of refractive index μ_2. If ξ' and η' refer to the principal axes of the $(\nu+2)$th layer, then using Eq. (1) the reflected components are

$$\begin{bmatrix}\xi'\\ \eta'\end{bmatrix}=-\frac{\beta\Delta\mu}{2\mu}\begin{bmatrix}i\\ 1\end{bmatrix}\exp[i\{(\nu+1)\beta-\phi_{\nu+1}\}]$$

$$=-iq\begin{bmatrix}1\\ -i\end{bmatrix}\exp[i\{(\nu+1)\beta-\phi_{\nu+1}\}],$$

where $\Delta\mu=\mu_1-\mu_2$, $2\mu=\mu_1+\mu_2$ and $|q|=\beta\,\Delta\mu/2\mu$. We make the approximation here that $\sin\beta\approx\beta$, since β is assumed to be very small. Transforming back to OX, OY, the reflected wave on reaching the surface of the liquid crystal will be

$$\begin{bmatrix}X\\ Y\end{bmatrix}=-iq\begin{bmatrix}1\\ -i\end{bmatrix}\exp[i\{(2\nu+3)\beta-2\phi_{\nu+1}\}],$$

which represents a right circular vibration travelling in the negative direction of OZ. Clearly the phase difference between this wave and the one reflected at the boundary between the first and second layers is $\exp[2i(\nu\beta-\phi_\nu)]$. When $\lambda=\mu P$, we have $2\pi\mu p/\lambda=\beta$ and $\phi_\nu=\nu\beta$ (since $np=P$ and $n\beta=2\pi$, where n is the number of layers per turn of the helix). Hence the phase factor $\exp[2i(\nu\beta-\phi_\nu)]$ becomes unity irrespective of the value of ν, and there results a strong interference maximum. On the other hand, for a left-handed structure, β is negative and $(\nu\beta-\phi_\nu)$ does not vanish.

The reflection coefficient $-iQ$ per turn of the helix (neglecting multiple reflections within the n layers) is then $-inq$ and the spectral width of reflection from a thick specimen[1,4]

$$\frac{Q\lambda_0}{\pi}=\frac{n\beta\Delta\mu\lambda_0}{2\pi\mu}=P\Delta\mu$$

in agreement with the de Vries theory.

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