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### Spectral Width of Reflection from a Cholesteric Liquid Crystal

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## Spectral Width of Reflection from a Cholesteric Liquid Crystal.

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The aim of this note is to rectify an error in the expression for the spectral width of reflection  $\Delta\lambda$  from a cholesteric liquid crystal derived in a previous paper.<sup>1</sup> The correct derivation presented here leads to a value of  $\Delta\lambda$  in agreement with the de Vries theory.<sup>2</sup>

We first write down the formulae for reflection when light is incident normally on the surface of a non-absorbing anisotropic crystal.<sup>3</sup> Let  $\mu_0$  be the refractive index of the first medium from which light is incident and  $\mu_1, \mu_2$  the principal indices of the anisotropic crystal. If the incident light is of unit amplitude and linearly polarized at an angle  $\theta$  with respect to the principal axis for which the refractive index is  $\mu_1$ , the reflected light will consist of two vibrations linearly polarized along the two principal axes:

$$\left. \begin{aligned} E_1 &= -\frac{\mu_1 - \mu_0}{\mu_1 + \mu_0} \cos \theta \\ E_2 &= -\frac{\mu_2 - \mu_0}{\mu_2 + \mu_0} \sin \theta \end{aligned} \right\} \quad (1)$$

Now, the cholesteric structure is regarded as a pile of thin birefringent layers, the principal axes of the successive layers turned through a small angle  $\beta$ . Let the principal axes of the first layer be along  $OX, OY$ . If the structure is right-handed, i.e.,  $\beta$  is positive, it can be shown<sup>4</sup> that right circular light incident normal to the layers is reflected without change of sense of circular polarization when  $\lambda_0 = \mu P$ , where  $P$  is the pitch,  $\mu$  the refractive index and  $\lambda_0$  the wavelength in

vacuum. To calculate the reflection coefficient at the boundary between the  $(\nu+1)$ th and  $(\nu+2)$ th layers, we resolve the incident light vector along the principal axes of the  $(\nu+1)$ th layer which are inclined at an angle  $(\nu+1)\beta$  with respect to  $OX, OY$ . The resolved components are<sup>4</sup>

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \begin{bmatrix} 1 \\ i \end{bmatrix} \exp[i\{(\nu+1)\beta - \phi_{\nu+1}\}],$$

where  $\phi_{\nu+1} = 2\pi\mu(\nu+1)p/\lambda$ , where  $p$  is the thickness of each layer. At the boundary, the  $\xi$  vibration emerges from a medium of refractive index  $\mu_1$  and the  $\eta$  vibration from a medium of refractive index  $\mu_2$ . If  $\xi'$  and  $\eta'$  refer to the principal axes of the  $(\nu+2)$ th layer, then using Eq. (1) the reflected components are

$$\begin{aligned} \begin{bmatrix} \xi' \\ \eta' \end{bmatrix} &= -\frac{\beta\Delta\mu}{2\mu} \begin{bmatrix} i \\ 1 \end{bmatrix} \exp[i\{(\nu+1)\beta - \phi_{\nu+1}\}] \\ &= -iq \begin{bmatrix} 1 \\ -i \end{bmatrix} \exp[i\{(\nu+1)\beta - \phi_{\nu+1}\}], \end{aligned}$$

where  $\Delta\mu = \mu_1 - \mu_2$ ,  $2\mu = \mu_1 + \mu_2$  and  $|q| = \beta\Delta\mu/2\mu$ . We make the approximation here that  $\sin\beta \approx \beta$ , since  $\beta$  is assumed to be very small. Transforming back to  $OX, OY$ , the reflected wave on reaching the surface of the liquid crystal will be

$$\begin{bmatrix} X \\ Y \end{bmatrix} = -iq \begin{bmatrix} 1 \\ -i \end{bmatrix} \exp[i\{(2\nu+3)\beta - 2\phi_{\nu+1}\}],$$

which represents a right circular vibration travelling in the negative direction of  $OZ$ . Clearly the phase difference between this wave and the one reflected at the boundary between the first and second layers is  $\exp[2i(\nu\beta - \phi_\nu)]$ . When  $\lambda = \mu P$ , we have  $2\pi\mu p/\lambda = \beta$  and  $\phi_\nu = \nu\beta$  (since  $np = P$  and  $n\beta = 2\pi$ , where  $n$  is the number of layers per turn of the helix). Hence the phase factor  $\exp[2i(\nu\beta - \phi_\nu)]$  becomes unity irrespective of the value of  $\nu$ , and there results a strong interference maximum. On the other hand, for a left-handed structure,  $\beta$  is negative and  $(\nu\beta - \phi_\nu)$  does not vanish.

The reflection coefficient  $-iQ$  per turn of the helix (neglecting multiple reflections within the  $n$  layers) is then  $-inq$  and the spectral width of reflection from a thick specimen<sup>1,4</sup>

$$\frac{Q\lambda_0}{\pi} = \frac{n\beta\Delta\mu\lambda_0}{2\pi\mu} = P\Delta\mu$$

in agreement with the de Vries theory.

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